Homework #5 (10 points) - Show all work on the following problems:

Problem 1 (2 points): Find the average potential over a spherical surface of radius R due to a point charge located inside the sphere (not at the center).

Problem 2 (2 points): In 1-d, the functional form of the general solution to Laplace's equation is $V(x) = mx + b$.

2a (1 point): Find the functional form of the general solution to Laplace's equation in 3-d spherical coordinates for the case where V only depends on the radial coordinate *r*.

2b (1 point): Find the functional form of the general solution to Laplace's equation in 3-d cylindrical coordinates for the case where V only depends on the radial coordinate *s*.

Problem 3 (2 points): Consider an infinite grounded conducting plane with two charges above the plane: $-2q$ at height d, and $+q$ and height $3d$. Use image charges to determine the force on the upper charge (+q).

Problem 4 (4 points): Consider a point charge q at a distance a from the center of a grounded conducting sphere of radius R (with $a > R$), as in Example 3.2 in Griffiths.

4a (1 point): Use the law of cosines to show that you can write

$$V(r,\theta) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{r^2+a^2-2racos\theta}} - \frac{q}{\sqrt{R^2+(\frac{ra}{R})^2-2racos\theta}}\right]$$

4b (1 point): Use the boundary conditions on the electric field (and thus the normal derivative of V) at the surface of the sphere to find the induced surface charge density σ on the sphere, as a function of θ.

4c (1 point): Integrate the charge density over the surface of the sphere to find the total induced charge.

4d (1 point): Calculate the energy of this configuration by determining the energy required to bring the charge q from infinity.